Chapter 17: Probability Models

Bernoulli Trials:

A situation is called a ***Bernoulli trial*** if it meets the following criteria:

- There are only two possible outcomes (categorized as _______________ or _______________) for each trial
- The probability of success, denoted ______, is the same for each trial
- The trials are _________________________________

  (Note that the independence assumption is violated whenever we sample without replacement, but is overridden by the __________ condition. As long as we don't sample more than 10% of the population, the probabilities don't change enough to matter.)

1. A new sales gimmick has 30% of the M&M's covered with speckles. These "groovy" candies are mixed randomly with the normal candies as they are put into the bags for distribution and sale. You buy a bag and remove candies one at a time looking for the speckles.

   a. Is this situation a Bernoulli trial? Explain.

   b. What's the probability that the first speckled candy is the fourth one we draw from the bag?

   c. What's the probability that the first speckled candy is the tenth one?

   d. Write a general formula.

   e. What's the probability we find the first speckled one among the first three we look at?

   f. How many do we expect to have to check, on average, to find a speckled one?

Geometric Distributions:

Suppose the random variable X = the number of trials required to obtain the first success. Then X is a _____________________ _____________________ _____________________ if:

1. There are only two outcomes: _____________________ or _____________________.
2. The probability of success p is _____________________ for each observation.
3. The n observations are _____________________.
4. The variable of interest is the ____.

Because *n* is not fixed there could be an infinite number of X values. However, the probability that *X* is a very large number is more and more unlikely. Therefore the probability histogram for a geometric distribution is always ____.

If *X* is a geometric random variable, it is said to have a _______________________________________, and is denoted as _______________.

The expected value (mean) of a geometric random variable is ______________.

The standard deviation of a geometric random variable is ___________________.

The probability that X is equal to *x* is given by the following formula:

2. Refer back to the M&M's distribution in problem 1.

   a. What's the probability that we'll find two speckled ones in a handful of five candies?

   b. List all possible combinations of exactly two speckled M&M's in a handful of five candies.

Binomial Distributions:

Suppose the random variable *X* = the number of successes in *n* observations.

Then *X* is a ____________________ ____________________ ____________________ if:

1. There are only two outcomes: ____________________ or ____________________.
2. The probability of success *p* is ____________________ for each observation.
3. The *n* observations are ____________________.
4. There is a ____ *n* of observations.

If *X* is a binomial random variable, it is said to have a _______________________________________, and is denoted as _______________.

The expected value (mean) of a binomial random variable is ______________.

The standard deviation of a binomial random variable is ___________________.

The ____________________ ____________________ ____________________ (or ___________) assigns a probability to each value of *X*.

The ____________________ ____________________ ____________________ (or____________) calculates the sum of the probabilities up to *X*.

Example: Suppose each child born to Jay and Kay has probability 0.25 of having blood type O. If Jay and Kay have 5 children, what is the probability that exactly 2 of them have type O blood?

Let X = ____.

1. There are only two outcomes: success (_______________) or failure (_______________).
2. The probability of success (_______________) is _______________ for each of the ________ observations.
3. Each of the 5 observations is ____________________, since one child's blood type will not influence the next child's blood type.
4. There is a fixed number of observations: __________.

So X is a ____.

The following table shows the probability distribution function (__________) for the binomial random variable, X.

x	0	1	2	3	4	5
$P(X = x)$	0.2373	0.3955	0.2637	0.0879	0.0146	0.001

P(X = 0) = P( ) = ($)^5$ = 0.2373

Binompdf ( ) = 0.2373
Binompdf ( ) = 0.3955
Binompdf ( ) = 0.2637
Binompdf ( ) = 0.0879
Binompdf ( ) = 0.0146
Binompdf ( ) = 0.0010

Construct a histogram of the p.d.f. using the window X_1 [0, 6] and $Y_{0.1}$ [0, 1].

The following table shows the cumulative distribution function (__________) for the binomial random variable, X.

x	0	1	2	3	4	5
$P(X=x)$	0.2373	0.3955	0.2637	0.0879	0.0146	0.001
$P(X \le x)$						

Binomcdf ( ) = 0.2373
Binomcdf ( ) = 0.6328
Binomcdf ( ) = 0. 8965
Binomcdf ( ) = 0. 9844
Binomcdf ( ) = 0. 999
Binomcdf ( ) = 1

Construct a histogram of the c.d.f. using the window X_1 [0, 6] and $Y_{0.1}$ [0, 1].

1. Suppose I have a group of 4 students and I want to choose 1 of them as a volunteer. In how many ways can I choose 1 out of 4 students?

   Call this "____________________." There are __________ ways.

2. Suppose I have a group of 4 students and I want to choose 2 of them as volunteers. In how many ways can I choose 2 out of 4 students?

   Call this "____________________." There are __________ ways.

3. Suppose I have a group of 5 students and I want to choose 1 of them as a volunteer. In how many ways can I choose 1 out of 5 students?

   Call this "____________________." There are __________ ways.

4. Suppose I have a group of 5 students and I want to choose 3 of them as a volunteer. In how many ways can I choose 3 out of 5 students?

   Call this "____________________." There are __________ ways.

5. Suppose I have a group of 20 students and I want to choose 4 of them as a volunteer. In how many ways can I choose 4 out of 20 students?

SSSSFFFFFFFFFFFFFFFF
FSSSSFFFFFFFFFFFFFFF
FSFFFFSFFFSFFFFFFFSF
… and so on…

Call this "____________________." There are __________ ways.

There is a mathematical way to count the total number of ways to arrange *k* out of *n* objects. This is called **"_________________"** or the ____.

The binomial coefficient is the number of ways to arrange *k* successes in *n* observations.

It is written __________ and is called "___________________."

The value of "*n* choose *k*" is given by the formula:

Example: "5 choose 2"

So there are __________ ways to arrange 2 out of 5 objects.

Think of this as flipping a coin 5 times and getting 2 heads. In how many ways can that happen? It can happen in __________ ways.

If X is a binomial random variable with parameters *n* and *p*, then

$P\left(X = x\right) =$