

Roots of Real Numbers and Radical Expressions

Definition of n^{th} Root

Square Root

Notation

$$\sqrt[4]{81}$$

Principal Root

Ex: $\sqrt{64}$
 $-\sqrt{64}$
 $\pm\sqrt{64}$

| The n^{th} root of b | | | | $\sqrt[n]{b}$ |
|---------------------------------|---------|---------|---------|---------------|
| n | $b > 0$ | $b < 0$ | $b = 0$ | |
| | | | | |
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Examples:

1. $\pm\sqrt{169x^4}$

2. $-\sqrt[4]{(8x)^3}$

3. $\sqrt[3]{125x^6}$

4. $\sqrt[3]{-m^3n^3}$

Taking nth roots of variable expressions - Using absolute value signs

Examples:

1. $\sqrt[4]{(an)^4}$

2. $\sqrt[6]{(xy^2)^6}$

3. $\sqrt{x^6}$

4. $\sqrt[6]{(3 - y^2)^{18}}$

Product Property of Radicals

Examples:

1. $\sqrt{30a^{34}}$

2. $\sqrt{54x^4y^5z^7}$

3. $\sqrt[3]{54a^3b^7}$

4. $\sqrt{60xy^3}$

Quotient Property of Radicals

Examples:

1. $\sqrt{\frac{7}{16}}$

2. $\sqrt{\frac{32}{25}}$

3. $\frac{\sqrt{48}}{\sqrt{3}}$

4. $\sqrt{\frac{45}{4}}$

Rationalizing the denominator

Ex: Simplify $\sqrt{\frac{5}{3}}$

Simplest Radical Form

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Examples:

1. $\sqrt{\frac{5}{4}}$

2. $\frac{20\sqrt{8}}{2\sqrt{2}}$

3. $\frac{5}{2\sqrt{2}}$

4. $4\sqrt{\frac{5}{7x}}$

Adding radicals – we can only combine terms with radicals if we have like radicals

Example: Simplify $6\sqrt{7} + 5\sqrt{7} - 3\sqrt{7}$

Examples:

$$1. \ 2\sqrt{3} + 5 + 7\sqrt{3} - 2$$

$$2. \ 5\sqrt{6} - 3\sqrt{24} + \sqrt{150}$$

Multiplying radicals

- *Distributing* – Simplify $\sqrt{3}(\sqrt{2} + 4\sqrt{3})$

- *FOIL* – Simplify $\sqrt{3} \times \sqrt{5} + \sqrt{2} \times \sqrt{3}$

Examples:

$$1. \ (2\sqrt{3} + 4\sqrt{5})(\sqrt{3} + 6\sqrt{5})$$

$$2. \ (5\sqrt{4} + 2\sqrt{7})(5\sqrt{4} - 2\sqrt{7})$$

Conjugates

Examples:

$$1. \frac{\sqrt{3} + 2}{\sqrt{3} - 5}$$

$$2. \frac{1 + 2\sqrt{5}}{6 - \sqrt{5}}$$