Trigonometry
Graphing Trig Functions with Transformations

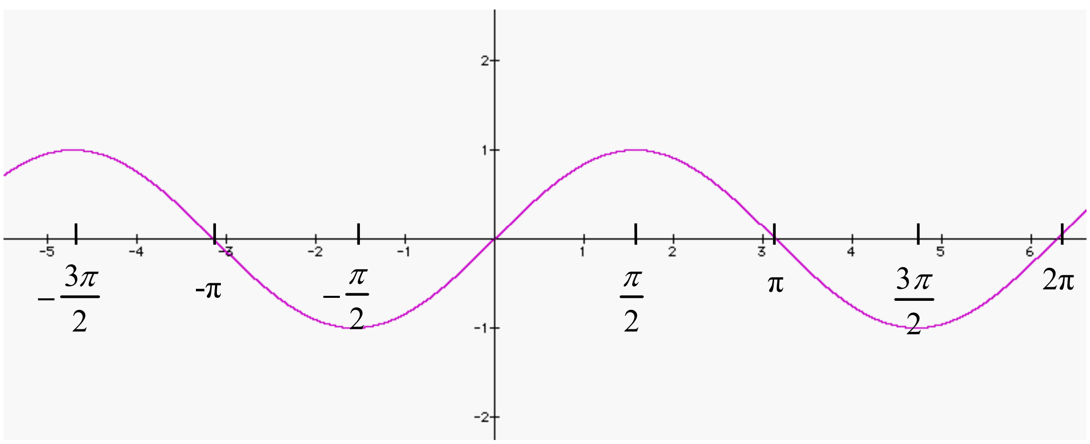
Let's start with an example: Graph $y = 3\sin(4\theta - \pi)$.

***Where Do I Start?***

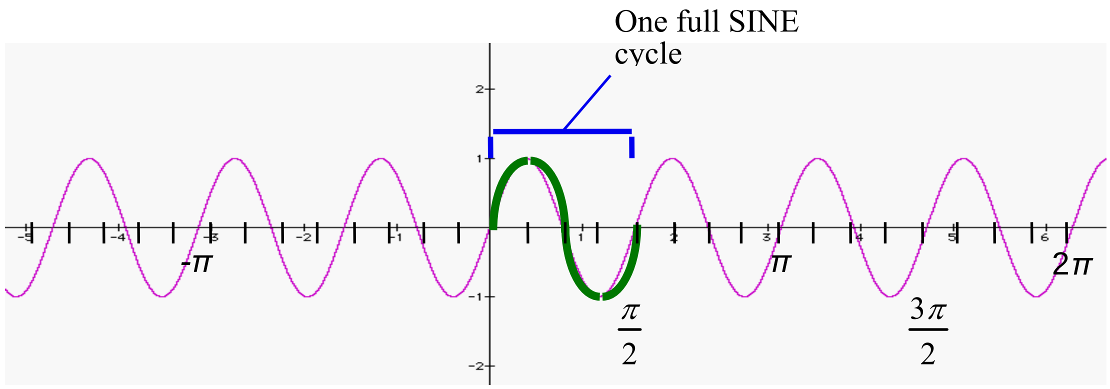
1. List the *Amplitude*, *Period, Phase Shift, Vertical Shift*, and any *Reflections*:

   $Amplitude = 3$
   $Period = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$
   $Phase\ Shift = -\dfrac{-\pi}{4} = \dfrac{\pi}{4}$
   *Vertical Shift* = NONE
   *Reflections*: NONE

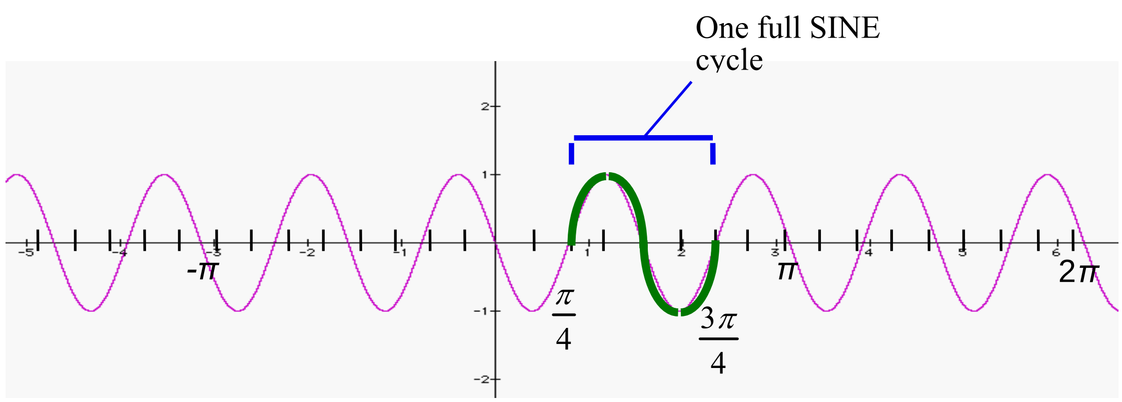
2. Graph the *Parent Function*. Make sure you know exactly where the graph crosses the x-axis and reaches its max and min values! The parent SINE function crosses the x-axis at 0, π, 2π, etc. and reaches its max y-value at π/2 and its min y-value at 3π/2. Trig functions are cyclical, so this pattern repeats every 2π.



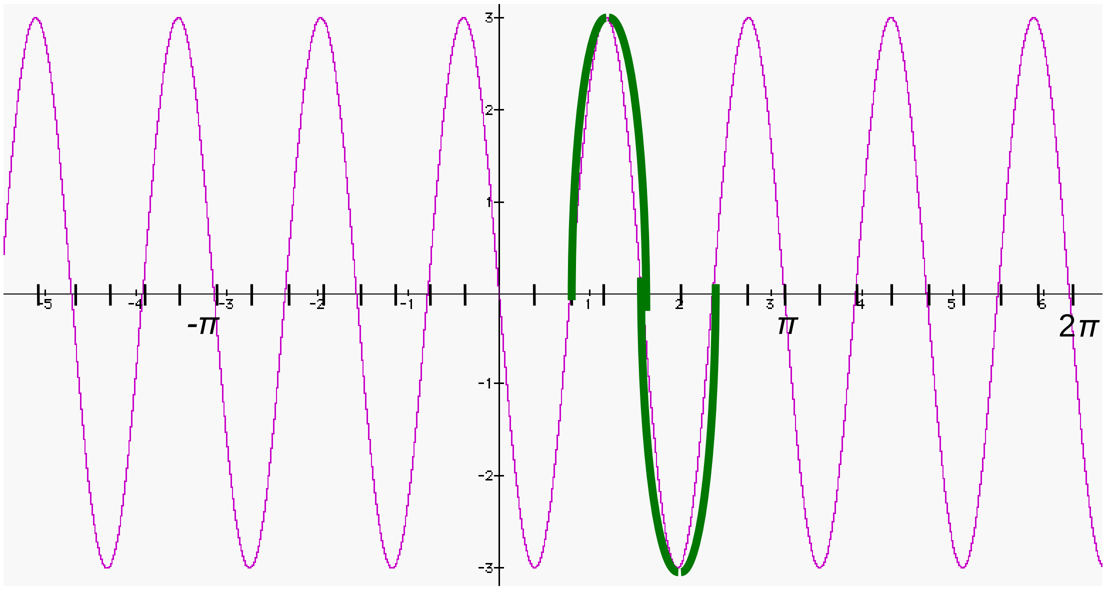
3. Graph the function with the new *Period*. If the period is not different from the parent function, then skip to the next step. In this problem, the period is π/2. Consider that one period starts at 0, then one full SINE cycle will end at π/2. It will reach the max at π/8, another zero at π/4, and the min at 3π/8, etc.




4. Graph the function with the new *Phase Shift*. Take the graph with the new period, and shift the graph left or right. Remember, the direction ends up being right if $c < 0$ and left if $c > 0$. Our phase shift is $\frac{\pi}{4}$ (to the RIGHT) so one period of the graph will go from $\frac{\pi}{4}$ to $\frac{3\pi}{4}$.



5. Graph the function with the new *Amplitude*. Take the graph with the new period and phase shift, and apply the new amplitude. The amplitude of our graph is 3.




6. Graph the function with any *Vertical Reflections*. Take the graph with the new period, phase shift and amplitude, and apply the reflection. Our graph does not have any reflections, so we will not draw a new graph.

7. Finally, graph the function with the new *Vertical Shift*. Our function does not have a vertical shift, so the graph in the previous step is our final graph.

***Your graph should be clear enough to answer questions pertaining to the graph.***

a) Name a point where the graph reaches its maximum height. $\left(\frac{3\pi}{8}, 3\right)$

b) Name a point where the graph reaches its minimum height. $\left(\frac{5\pi}{8}, -3\right)$

c) Name three points where the graph is at equilibrium. $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$ & $\left(\frac{3\pi}{4}, 0\right)$

d) How often is the graph at equilibrium? Every $\frac{\pi}{4}$ radians.

e) How often does the graph reach a minimum point? Every $\frac{\pi}{2}$ radians.

*It is important to apply the transformations, use the order of operations, start inside the parentheses, applying the coefficient (period change) before the constant (phase shift), multiplying before adding. Then do the same thing outside the parentheses, applying the coefficient (amplitude change) before the constant (vertical shift), multiplying before adding.*

**Example 2**: Graph $y = -5\tan\left(\frac{\theta}{3} + 90°\right)$.

1. *Amplitude* = NONE (only SINE and COSINE have amplitude)

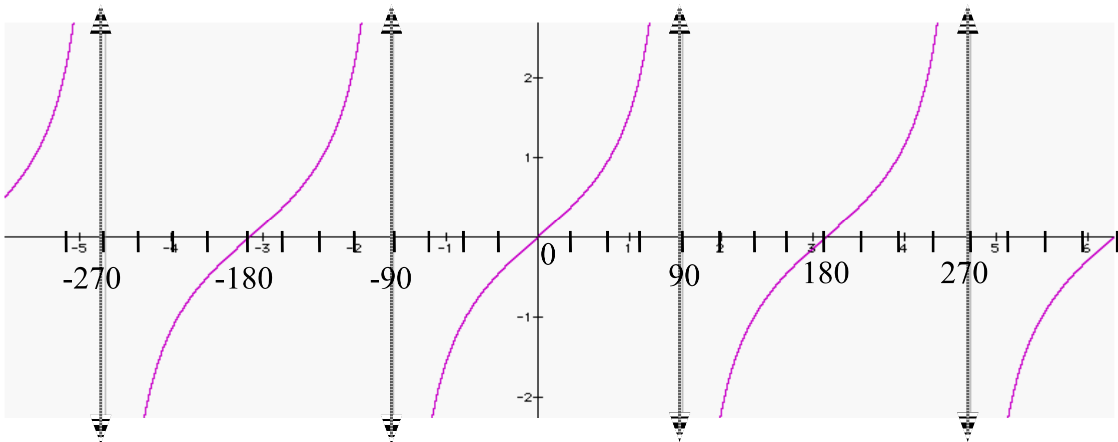
   $$Period = \frac{180°}{\frac{1}{3}} = 180° \cdot \frac{3}{1} = 540°$$

   $$Phase\ Shift = -\frac{90°}{\frac{1}{3}} = -90° \cdot \frac{3}{1} = -270°$$

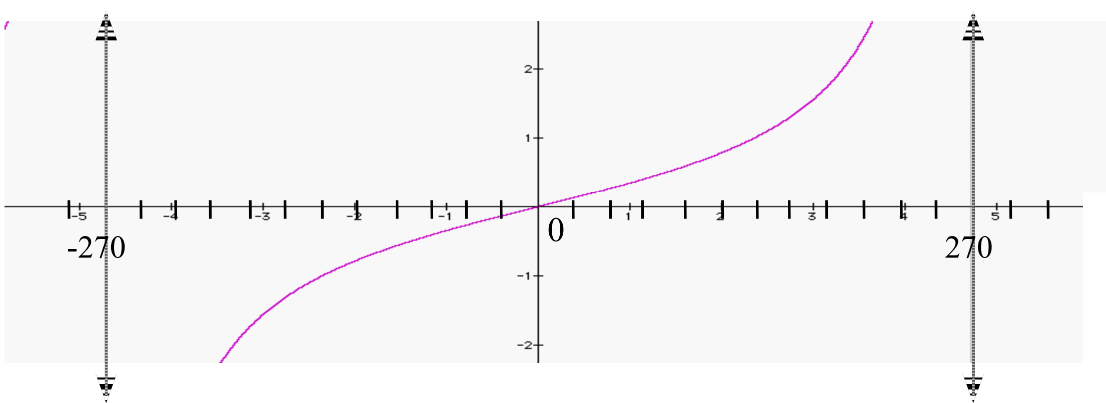
   *Vertical Shift* = NONE

   *Reflections*: Vertical

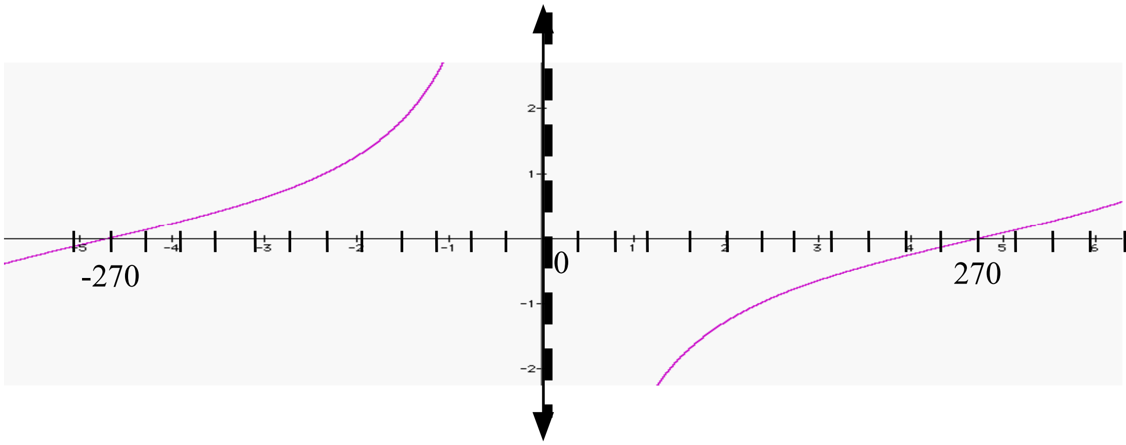
2. Parent Function: $y = \tan\theta$. Each curve is 180° wide with vertical asymptotes between.




3. New Period: 540°. On the parent, one curve goes from –90° to 90° for a period of 180°. On the new graph the period is 540°, so one curve will go from –270° to 270°.



4. New Phase Shift: -270° (to the LEFT). So the curve that went from –270° to 270° now goes from –540° to 0° and the next curve will go from 0° to 540° with an asymptote between. The graph below still shows just one cycle of the graph, from –270° to 270°. The asymptote is at 0°.




5. There is no amplitude, but the coefficient, -5, does have an effect on the graph ... what is the effect? Also, since the coefficient is negative, there is a vertical reflection: