Trigonometry
Graphing Trig Functions: Amplitude, Period and Phase Shift

Terminology

Parent Function

You should be familiar with the Parent Functions of the trig graphs.

$y = \sin\theta$ $\quad$ $y = \csc\theta$

$y = \cos\theta$ $\quad$ $y = \sec\theta$

$y = \tan\theta$ $\quad$ $y = \cot\theta$

Transformed Function

For the purpose of this lesson, we will use the following model to discuss the transformation of parent trig functions. (The sine function can be replaced by any of the other trig functions.)

$$y = a\sin(b\theta + c) + d$$

Amplitude

The term **amplitude** is only applied to the sine and cosine graphs. Amplitude is the absolute value of the maximum displacement of $\sin\theta$ or $\cos\theta$ from equilibrium. The parent functions of sine and cosine are both at equilibrium at y = 0 and they both achieve maximum y-values of 1 and minimum y-values of -1. The amplitudes of $y = \sin\theta$ and $y = \cos\theta$ are 1. The amplitude changes, however, when a coefficient is multiplied by the trig function.

In the function $y = a\sin(b\theta + c) + d$ **,** the **AMPLITUDE = |a|.**

Period

All of the trig functions repeat. The **period** is the length of one cycle of a function. The parent functions of sine, cosine, secant & cosecant have periods of 360° or 2π radians. The parent functions of tangent & cotangent have periods of 180° or π radians. The length of the period changes, however, when a coefficient is multiplied by θ in the trig function.

In the function $y = a\sin(b\theta + c) + d$ **,** the **PERIOD** $= \frac{360°}{b}$ **or** $\frac{2\pi}{b}$**.**

(This is also true for cos, sec & csc.)

In the function $y = a\tan(b\theta + c) + d$ **,** the **PERIOD** $= \frac{180°}{b}$ **or** $\frac{\pi}{b}$**.**

(This is also true for cot.)

Phase Shift

Phase shift is the translation of a trig function to the left or right. The parent functions, sin, cos, tan, etc., are not shifted. A phase shift is applied when a constant is added inside the trig function.

In the function $y = a\sin(b\theta + c) + d$ **,** the **PHASE SHIFT** $= -\frac{c}{b}$**.**

(This is also true for the other five trig functions.)

Note: If $c > 0$, then the shift is to the left.
If $c < 0$, then the shift is to the right.

Vertical Shift

Vertical Shift is the translation of a trig function up or down. The parent functions, sin, cos, tan, etc., are not shifted. A vertical shift is applied when a constant is added outside the trig function.

In the function $y = a\sin(b\theta + c) + d$ **,** the **VERTICAL SHIFT = d.**
(This is also true for the other five trig functions.)

Other Transformations

Vertical Reflection

A vertical reflection is when the graph is reflected across the *x*-axis, like a mirror image being flipped upside-down. This happens when the coefficient '*a*' is negative.

Horizontal Reflection

A horizontal reflection is when the graph is reflected across the y-axis, flipping the graph left-right. This happens whent he coefficient '*b*' is negative.

Multiple Transformations in One Function

One trig function is likely to have more than one transformation being applied at a time. Consider each transformation one step at a time to determine the overall changes to the parent graph:

Example For the function $y = 4\sin(2\theta + 90°) - 6$ **:**

$a = 4,\ b = 2,\ c = 90°,\ d = -6$

the amplitude is 4,
the period is $\frac{360°}{2} = 180°$,
the phase shift is $-\frac{90°}{2} = -45°$ (a shift to the left),
the vertical shift is –6.

Example For the function $y = 4\tan\left(\frac{\theta}{4} - \frac{\pi}{6}\right)$ **:**

$a = 4,\ b = \frac{1}{4},\ c = -\frac{\pi}{6},\ d = 0$

there is no amplitude, tangent increases and decreases without bound,
the period is $\frac{\pi}{\frac{1}{4}} = \pi \cdot \frac{4}{1} = 4\pi$,
the phase shift is $-\frac{-\frac{\pi}{6}}{\frac{1}{4}} = \frac{\pi}{6} \cdot \frac{4}{1} = \frac{2\pi}{3}$ (a shift to the right),
the vertical shift is 0.

Example For the function $y = -3\cos\left(\theta - \frac{\pi}{2}\right) + 1$ **:**

$a = -3,\ b = 1,\ c = -\frac{\pi}{2},\ d = 1$

the amplitude is 3,
the period is $\frac{2\pi}{1} = 2\pi$,
the phase shift is $-\frac{-\frac{\pi}{2}}{1} = \frac{\pi}{2} = \frac{\pi}{2}$ (a shift to the right),
the vertical shift is 1.

This graph is also reflected vertically since a = -3.

Try These

1. State the amplitude, period, phase shift and vertical shift for each of the following.

   a) $y = 3\cos(3\theta)+1$
   b) $y = -5\sin(6\theta)$
   c) $y = 4\tan(2\theta-\pi)$
   d) $y = 24\sec(2\theta+150°)$
   e) $y = -3\cot\left(\frac{\theta}{4}+\frac{\pi}{8}\right)$

2. Write an equation of a sine function with amplitude 3, period 720°, and phase shift 90° (to the right).

3. Write an equation of a tangent function with period 2π and phase shift of $-\frac{\pi}{4}$ (to the left).